ME-221
PROBLEM SET 4

Problem 1

Consider the system defined by the following state equations:

$$\dot{x_1} = x_1 + x_2 - (u_1 + u_2) \qquad x_1(0) = 1$$
$$\dot{x_2} = x_1^2 - (x_2 - 1)^2 + x_1 x_2 - u_1^2 - u_2 \qquad x_2(0) = 1$$
$$y_1 = x_1(1 + x_2) + u_1$$
$$y_2 = x_1 + x_2 - u_2$$

Linearize the model around a stationary point corresponding to $\bar{u}_1 = \bar{u}_2 = 1$ and for positive values of $\bar{x}_1$ and $\bar{x}_2$. Obtain a state-space representation for the linearized system.

Problem 2

Consider the mechanical system shown in Figure 1. The system is driven by an external force F applied to the mass in a direction perpendicular to the pendulum arm. The output of the system is the angular position θ and the moment of inertia is given by $J = ml^2$. The spring is in relaxed state when $\theta = 0$. Consider that the spring always stays horizontal.

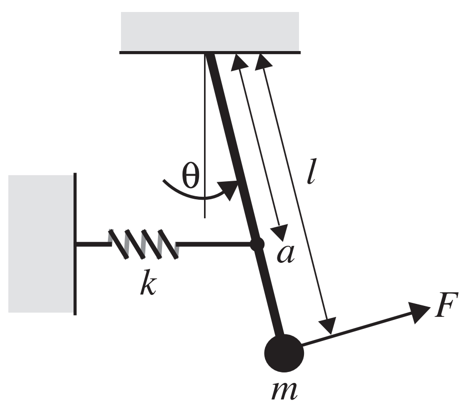


Figure 1: Pendulum system

a) Obtain a state-space representation for the system.
b) Linearize the model for small deformations around the vertical equilibrium position.

Problem 3

Figure 2 shows a magnetic ball that is levitated in air using an electromagnetic coil. The input and output of the system are the current i passing through the coil and the position x of the ball, respectively.

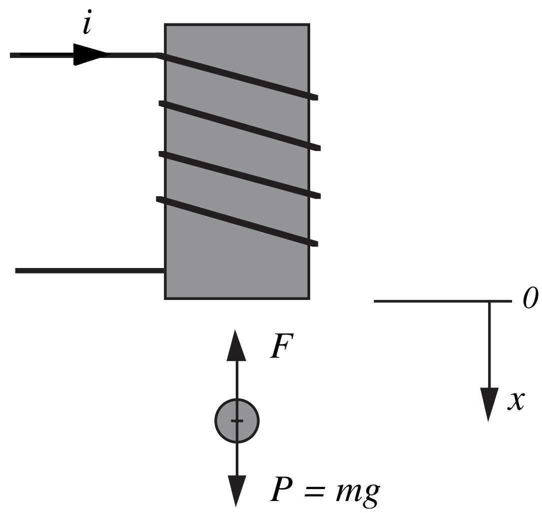


Figure 2: Magnetic levitation system

F depends on the distance x and the current i that is flowing through the wires according to the following formula:

$$F(x,i) = \frac{1}{2}\frac{L}{(1+x)^2}i^2$$

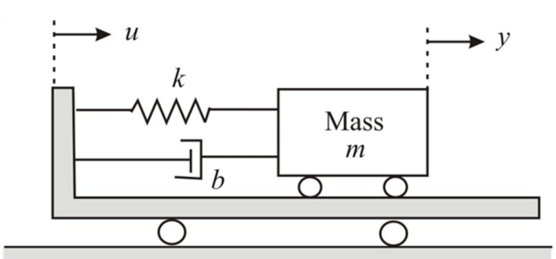
where L is an inductance term. The equation that governs the movement of the ball is given by:

$$m\ddot{x} = mg - F(x,i)$$

where m represents the mass of the ball and g is the gravitational acceleration.

a) Derive the state equations for the system
b) Obtain a linearized version of the state-space representation.

Problem 4



Consider the spring-mass-damper system mounted on a massless cart as shown on the left. Input $u(t)$ is the displacement of the cart. Output $y(t)$ is the displacement of mass m relative to the ground. The spring and viscous damping coefficients are denoted by k and b, respectively. Find the state-space representation of the system.